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THE ENTRANCE EFFECT OF LAMINAR FLOW OVER A BACKWARD-FACING STEP GEOMETRY

I. E. BARTON*

Division of Aerospace Engineering, Department of Engineering, University of Manchester, Oxford Road, Manchester M13 9PL, U.K.

SUMMARY

The study investigates the entrance effect for flow over a backward-facing step by comparing predictions that set the inlet boundary at various locations upstream of the sudden expansion. Differences are most significant in the sudden expansion region. If the geometry has an inlet channel, then shorter reattachment and separation lengths are predicted. Comparisons with experimental data indicate that better agreement is found using a long inlet channel, but only for low Reynolds numbers where the experimental error is less significant. For certain cases, predictions with a high expansion number are perturbed by the entrance effect more than low-expansion-number predictions; however, the effect is localized in the sudden expansion region. Channels with low expansion numbers always experience a greater entrance effect after some distance upstream and downstream of the sudden expansion. The boundary layer growth in the inlet channel was examined using a uniform inlet velocity profile.

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INTRODUCTION

Flow over a backward-facing step is fundamental in design and geometry and consequently it is found in a variety of engineering applications. The flow separation process caused by the sudden change in geometry has been used extensively in applications, usually in order to create a recirculation region or a sudden change in pressure. However, our understanding of this phenomenon is still incomplete even for laminar flow, despite the fact that laminar flow over the backward-facing step geometry of Armaly *et al.*[1] has become a classical numerical problem.

The majority of recent numerical studies investigating laminar flow over a backward-facing step similar to the configuration of Armaly *et al.*[1] have not considered the physics of flow in depth,[2–15] with the notable exceptions of Durst and Pereira,[7] Thangam and Knight,[10] Barton[14] and Gresho *et al.*[15] Durst and Pereira[7] and Gresho *et al.*[15] considered the growth of the recirculation regions with time; Gresho *et al.*[15] also consider the stability of flow for a high-Reynolds-number case. Thangam and Knight[10] studied the effect of the step height on the downstream flow. Barton[14] considered the effect of the viscous drag from the upper boundary.

* Correspondence to: I. E. Barton, Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, U.K.

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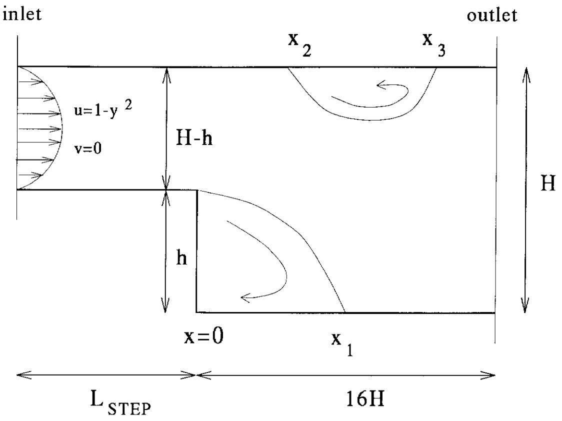

Figure 1. Backward-facing step geometry and recirculation regions

The present study is a continuation from the study by Barton[14] and focuses attention on how the sudden expansion affects the flow. This is achieved by analysing differences in the flow illustrated in Figure 1, where the geometry has an inlet channel of varying length and expansion number and the inlet flow profile is either a parabola as illustrated or a uniform profile. The differences in the numerical results that have a long inlet channel compared with no inlet channel and the boundary layer growth upstream of the sudden expansion are usually described as the 'entrance effect'. The entrance effect has not been previously studied for the present geometry despite the problem being considered a classical numerical benchmark,[16–18] although Han *et al.*[19] have applied artificial far-field boundary conditions upstream and downstream to the present flow configuration for a low Reynolds number using a vorticity–streamfunction methodology. Pollard[20] numerically studied the entrance effect for an axisymmetric expansion and found insignificant differences. The entrance effect for the present study is more significant because the sudden expansion region is not symmetrical.

NUMERICAL ANALYSIS

The governing equations for planar, steady incompressible flow can be described by simplified Navier–Stokes equations and the equation of continuity. These equations are expressed as

$$\frac{\partial \rho u^2}{\partial x}+\frac{\partial \rho uv}{\partial y}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right),$$
$$\frac{\partial \rho uv}{\partial x}+\frac{\partial \rho v^2}{\partial y}=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right),$$
$$\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}=0,$$

where ρ is the density and μ is the viscosity of the flow. The flow parameters are the inlet Reynolds number and the expansion number which are discussed later. The governing equations are solved in primitive form (u, v, p) using the 'semi-implicit method for pressure linked equations' (SIMPLE) methodology.[21] The numerical methodology used in the present study is very similar to the previous study[14] which followed the recommendations of Patankar.[22] The main difference is the application of the 'quadratic upwind interpolation for convection kinetic' (QUICK) differencing scheme[23] instead of the hybrid differencing scheme.[24] The QUICK scheme was found to be more successful in predicting grid-dependent results.

Non-uniform grid distributions were used for simulations with clustering near solid boundaries, especially around the step. Grid-independent results are presented where solutions used grids which spanned 120–240 grid points in the x-co-ordinate and 80–100 in the y-co-ordinate. The grid dependence of computations was tested using similarly clustered grids with 40×40, 80×60 and 100×80 grid points for the flow without an entrance channel; a similar grid dependence study has been carried out by Barton.[25] The differences are typically less than 1% for the finest grid results. The largest differences tend to occur for the separation length x_2: for some of the higher Reynolds numbers the differences between the two sets of results are of the order 2%. Comparisons were undertaken for all simulation cases with variation in expansion number and inlet Reynolds number.

The outlet condition was found by extrapolation; first-order and quadratic extrapolations recommended by Peric[26] were found to give fairly poor solutions in the outlet region. The outflow boundary condition developed and discussed by Barton[27] was applied. In short, the extrapolated velocities at the outlet are calculated using the following fit:

$$u = A + \frac{B}{x/\Delta x} + \frac{C}{(x/\Delta x)^2}.$$

In the extrapolation the four velocity positions upstream of the outflow boundary are used. The velocity position furthest away from the outflow boundary is used as the datum position for x; the velocity values at the other three positions are used for extrapolation calculation. The term Δx is the cell length adjacent to the outflow boundary. Therefore, if uniform cells are used near the exit region, the extrapolated velocity value is estimated by (using compass notation)

$$u_{\mathrm{EXIT}} = \frac{27u_{\mathrm{W}} - 12u_{\mathrm{WW}} + u_{\mathrm{WWW}}}{16}$$

and is then corrected to ensure that the overall flux is conserved. This formulation appears to reduce numerical errors near the outflow boundary; for further details see Reference 27. (The SIMPLE methodology does not require outlet pressure terms.) The superiority of this approach was tested by comparing truncated and 'long' outlet channel results at a variety of locations and inlet Reynolds numbers. The solutions presented in this study set the outlet boundary $16H$ downsteam of the sudden expansion, where H is the main channel height.

Similar to other numberical studies, either a fully developed parabola or a uniform u-velocity profile is prescribed at the inlet. No-slip boundary conditions are applied for the wall boundaries.

The u-velocity parabola profile deforms in the inlet channel because the effect of the sudden expansion travels upstream. The u-velocity speeds up along the lower boundary and slows down along the upper boundary of the inlet channel. Therefore the u-velocity profile deviates from a parabola in the inlet channel. The average deviation AD in the inlet channel is calculated using the following definition, where I and J are the inlet channel nodes and u^{PARABOLA} is the parabola u-velocity profile:

$$AD = \frac{\sum |U_{I,J} - u_J^{\mathrm{PARABOLA}}|}{\sum u_J^{\mathrm{PARABOLA}}}.$$

A pressure gradient forms across the inlet channel which forces flow away from the upper boundary; the pressure coefficient Cp_y can be used to examine this effect. The term Cp_y is defined as

$$Cp_y = \frac{h\partial p/\partial y}{\rho u_{\mathrm{INLET}}^2}.$$

RESULTS

Classical benchmark flow

Initially, the classical benchmark recommended by Gresho[18] is investigated, where the expansion number is fixed at $E = 2$ and a parabolic velocity profile is set at the inlet. The expansion number E is defined as the ratio of the main channel height to the inlet channel height. Predictions that have a very long inlet channel (over $10h$ long, where h is the step height) are compared with predictions using no inlet channel. For such a comparison exercise it is more appropriate to apply an artificial far-field boundary condition.[28] The benchmark sets the inlet Reynolds number to $Re = 800$, where the Reynolds number is defined using twice the inlet channel height, $2h$, the average inlet velocity u_{INLET} and the fluid's dynamic viscosity. This definition is the same as that of Armaly *et al.*[1] In the present study, unlike the benchmark, the Reynolds number is varied. The inlet Reynolds number was not varied above $Re = 800$, because the SIMPLE methodology can only successfully predict steady state flows. Gresho *et al.*[15] have established that the flow is steady at an inlet Reynolds number up to $Re = 800$ (where $E = 2$) but have not established when the flow becomes unstable. Furthermore, the experimental results of Armaly *et al.*[1] become transitional at $Re = 1200$; although this is an unreliable guide for when the flow becomes unsteady, it does suggest that a higher inlet Reynolds number range will be unsuccessful for the current investigation.

Benchmark results[29,30] (where $Re = 800$) are shown in Table I of reattachment and separation positions (non-dimensionalized with the step height) and are compared with the present results. The numerical results shown in the table are taken from the results that use the finest grid and best outlet treatment in their particular study. There is quite a variety of disagreement, although the present benchmark solutions (no-inlet-channel results) are in good agreement with Gartling.[29] The difference between the results that use a long inlet channel and no inlet channel is small in comparison with the various numerical results, but it should be emphasized that this comparison exercise uses the same grid distribution and outlet conditions in the main channel, so only upstream physical effects will cause any differences.

The largest values of AD and Cp_y at the sudden expansion occur for low Reynolds numbers, see Table II. This is in accordance with the extreme cases: a very low Reynolds number will force flow around the corner step without experiencing considerable separation, whereas a high-Reynolds-number flow will hardly experience the sudden expansion. The maximum downward v-velocity at the sudden expansion (non-dimensionalized with the $Re = 50$ case) for a geometry with a long inlet channel increases rapidly for low Reynolds numbers and then increases almost linearly for $Re > 300$; see Table II. The v-velocity results appear to indicate that the high-Reynolds-number cases are most affected by the entrance effect, contradicting the AD results. However, by non-dimensionalizing the maximum downward v-velocity with the mean inlet u-velocity (see Table II), it is shown that the low-Reynolds-number flows are most affected by the entrance effect. These results are analogous with a

Table I. Reattachment and separation positons of benchmark backward-facing step flow

	x_1	x_2	x_3
Gartling[27]	12·20	9·70	20·96
Betts and Sayma[28]	11·21	8·40	20·86
Srinivasan and Rubin[28]	12·44	10·18	20·50
Present, no entrance	12·03	9·64	20·96
Present, with inlet channel	11·51	9·14	20·66

Table II. Variation in inlet parameters for entrance channel geometry with Reynolds number

Re	AD	V_{MAX}/V_{MAX}^{50}	V_{MAX}/U_{INLET}	Cp_y
50	$8{\cdot}202\times10^{-2}$	1·000	0·1214	0·2676
100	$6{\cdot}063\times10^{-2}$	1·245	$7{\cdot}561\times10^{-2}$	0·1533
200	$3{\cdot}954\times10^{-2}$	1·391	$4{\cdot}224\times10^{-2}$	$6{\cdot}974\times10^{-2}$
300	$2{\cdot}936\times10^{-2}$	1·445	$2{\cdot}924\times10^{-2}$	$4{\cdot}337\times10^{-2}$
400	$2{\cdot}354\times10^{-2}$	1·483	$2{\cdot}251\times10^{-2}$	$3{\cdot}223\times10^{-2}$
500	$2{\cdot}039\times10^{-2}$	1·524	$1{\cdot}851\times10^{2}$	$2{\cdot}509\times10^{-2}$
600	$1{\cdot}805\times10^{-2}$	1·564	$1{\cdot}583\times10^{-2}$	$2{\cdot}052\times10^{-2}$
700	$1{\cdot}630\times10^{-2}$	1·603	$1{\cdot}391\times10^{-2}$	$1{\cdot}678\times10^{-2}$
800	$1{\cdot}484\times10^{-2}$	1·642	$1{\cdot}252\times10^{-2}$	$1{\cdot}370\times10^{-2}$

heat transfer study of Kondoh *et al.*,[13] where heat transfer effects were predicted to travel upstream of the step for low-Peclet-number cases.

The variations in the reattachment and separation lengths x_1, x_2 and x_3 (illustrated in Figure 1) with Reynolds number are summarized in Figure 2 for the long inlet channel and no inlet channel. The figure also shows the experimental data of Armaly *et al.*[1] The results indicate that the main reattachment length x_1 increases with Reynolds number with a slight non-linear trend. The non-linear trend is caused by the upper boundary where the flow is retarded by viscous effects for low Reynolds numbers and more significantly by the upper recirculation region for higher Reynolds numbers.[10,14] The figure shows that the geometry with a long inlet channel predicts shorter lengths x_1 and x_2; this is caused by the downward movement of flow at the inlet and the pressure gradient across the inlet channel. In both cases the difference is about $\frac{1}{2}h$ in length, although the difference for x_1 decreases for $Re < 300$. The fact that the differences are small for low Reynolds numbers appears to contradict the previous conclusion that low-Reynolds-number flows are most significantly affected by the entrance effect. This slight anomaly is explained by considering the inertia effects, for although the low Reynolds numbers are most affected by the entrance effect, it is localized in the sudden expansion

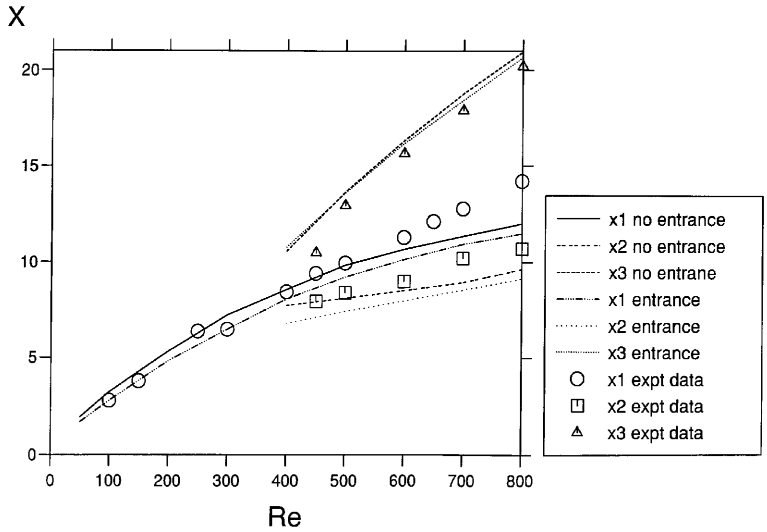

Figure 2. Variation in reattachment and separation lengths with Reynolds number

region because the inertia of the flow is too small to transport the effects downstream. The similar upper reattachment length x_3 for both geometries demonstrates how the entrance effect becomes gradually dissipated in the flow downstream of the sudden expansion. Figure 2 also indicates that better agreement is found for the low-Reynolds-number flows using an inlet channel with the experimental data of Armaly *et al.*[1] This is a satisfactory outcome as the low-Reynolds-number results are the most reliable, because three-dimensional experimental effects were observed for higher Reynolds numbers. The fact that better agreement is found for the high Reynolds numbers without using an inlet channel may be considered coincidental, although vortices or boundary layer growth from the side walls are likely to cuase a constriction in the flow and therefore will probably cause a longer lower reattachment length. In the inlet channel the average deviation from the parabolic u-velocity profile, AD, the pressure coefficient Cp_y and the maximum downward v-velocity (at a particular cross-section) decrease exponentially with distance upstream of the sudden expansion for the geometry with a long inlet channel. The terms AD, Cp_y and V_{MAX} start growing approximately two to three step heights upstream of the sudden expansion irrespective of the Reynolds number, which suggets an inlet channel four step heights long is sufficient for this problem.

Next, the behaviour of the flow with intermediate inlet channel lengths is considered for an expansion number $E = 2$ and a parabolic inlet velocity profile. The inlet channel length L_{STEP} is varied from zero to nine step heights upstream of the sudden expansion. Slightly increasing the inlet channel length causes the reattachment and separation positions to sharply decrease in size by the same amount. The linear perturbation is caused by the flow speeding up along the lower inlet boundary, whereas the main body of the flow is not greatly affected and therefore the flow structure is essentially the same. The main body of the flow starts to experience the acceleration effects of the step and the drag from the upper boundary as the inlet channel is made longer. The perturbations of the reattachment and separation lengths from the $L_{\mathrm{STEP}} = 0$ case are shown in Figure 3 for $Re = 200$, 400 and 600. The higher-Reynolds-number results experience virtually no changes if the inlet channel is at least three step heights upstream of the sudden expansion. The $Re = 400$ and 600 results clearly demonstrate in Figure 3 how initially increasing the inlet channel by a small amount changes all the reattachment and separation positions by the same amount, but as the inlet channel length gets

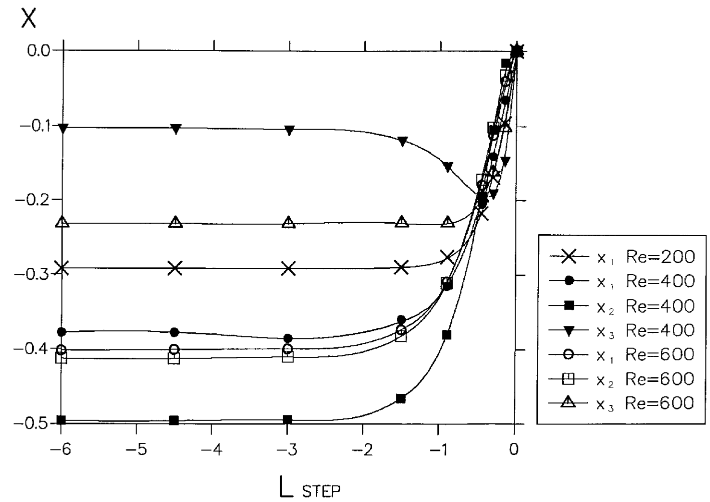

Figure 3. Perturbation of reattachment and separation lengths with various inlet boundary positions for $Re = 200$, 400 and 600

longer, the structure of the flow changes and the changes in the reattachment and separation positions are not necessarily monotonic. This demonstrates that there are two processes in the transition. By examining the flow, it was found that the first immediately establishes itself is a pressure gradient across the entrance which causes the flow to have a downward velocity, resulting in shorter reattachment and separation positions. The second process is deformation of the inlet profile, which requires a longer entrance channel to become a significant effect. The deformation of the inlet profile causes the structure of the flow to change, which also results in x_1 and x_2 moving upstream; however, as Figure 3 shows, this is not necessarily the case for x_3. The second process causes the adverse pressure gradient at the sudden expansion to be affected, with the result that a larger upper recirculation region is generated. The larger upper recirculation region tends to grow with the displacement of x_2, as viscous forces tend to dissipate differences further downstream. The significant factors are therefore the position of x_2 and the size of the upper recirculation region, which is observable in the $Re = 400$ case by the changes in x_3. The $Re = 600$ results show that the position of the reattachment length x_3 remains constant while the separation position x_2 moves upstream, causing a larger recirculation region. Compared with the $Re = 400$ case, the changes are smaller demonstrating how a high-Reynolds-number flow is less affected by the entrance effect.

Boundary layer growth

Pollard[20] investigated the behaviour of boundary layer growth in the inlet channel for an axisymmetric sudden expansion by imposing a uniform velocity profile at various locations upstream of the sudden expansion. Previously, Barton[14] used a uniform velocity profile set at the sudden expansion, in part to study the effect of viscous drag from the upper boundary. The present study uses Pollard's approach to study boundary layer growth for a sudden expansion of $E = 2$, which is same as the benchmark problem of Gresho.[18] The uniform velocity profile gradually deforms owing to the viscous action of the channel walls into the near-parabolic profile used in the benchmark. The average inlet velocity is set so that $Re = 800$ (the same as the benchmark), which means that the results that use a very long inlet channel should ultimately be similar to the benchmark solution.

The reattachment and separation positions using a uniform velocity profile are shown in Figure 4 for various inlet positions. The position of the inlet chanel is given by L_{STEP} in the figure. The figure also shows the benchmark results obtained in the present study; it is clearly seen that as the inlet channel is made longer, the reattachment and separation positions tend towards the benchmark results. Shorter channel lengths have a stronger pressure gradient across the inlet channel and a higher velocity near the separation point, causing a shorter lower reattachment length and smaller upper recirculation region.

Asymmetric boundary layer growth was found not to occur two step heights upstream of the sudden expansion for any inlet channel length. The boundary layer growth was also found to be initially symmetric even for short inlet channel lengths. Some boundary layer growth results are shown in Figure 5 for various inlet channel lengths, where the boundary layer thickness against upstream position x_{STEP} is plotted for the bottom and top boundary layers. The boundary layer thickness is defined as the distance from the boundary where $u/u_{MAX} = 0{\cdot}99$ and is non-dimensionalized by the step height.

The boundary layer thickness of the top boundary is larger than that of the bottom boundary near the sudden expansion, which is caused by the freestream conditions downstream of the lower boundary. Thus they allow the velocity to speed up along the lower boundary and force the flow to slow down along the upper boundary as previously discussed.

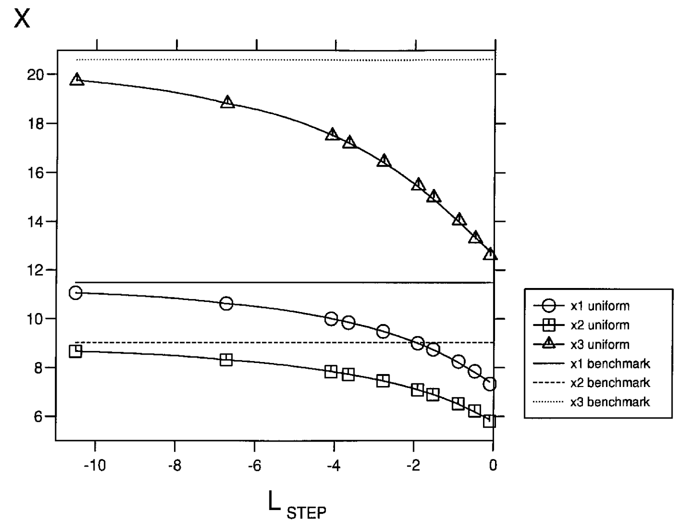

Figure 4. Reattachment and separation positions using uniform inlet velocity profile set at various upstream positions, and present benchmark results

Expansion number effect

Finally, the effect of expansion number is considered. Previous studies found that increasing the expansion number (from just above unity; $E = 1$ is an *open* backward-facing step flow) caused the behaviour of the flow to change from essentially an open backward-facing step flow to a wall-jet-type flow and the growth of the lower reattachment length had greater linearity for lower expansion numbers.[10]

An important issue in dealing with the effect of expansion number is how the length and velocity scales are defined.[14] For very low expansion numbers the average inlet velocity and the step height are reasonable scales because the flow behaves like an open backward-facing step. For higher

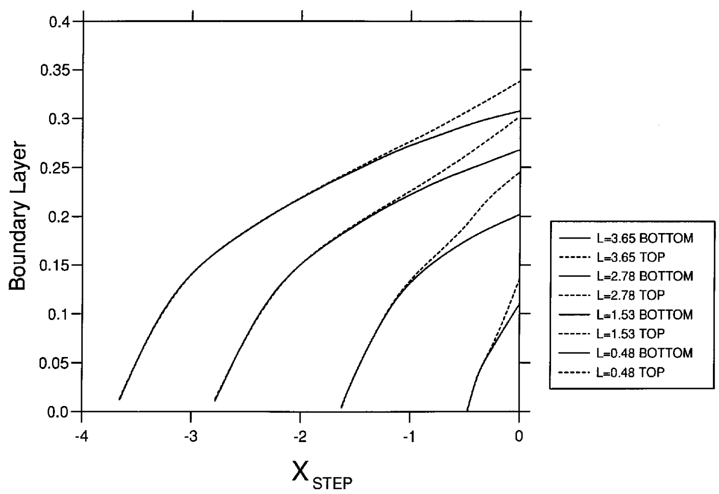

Figure 5. Variation in boundary layer thickness with position for various inlet boundary locations

expansion numbers the flow rapidly decelerates to u_{INLET}/E. Thus an appropriate alternative would be to use u_{INLET}/E as the velocity scale. A third possibility is to consider the total momentum entering the channel, in which case the average inlet velocity and the height of the inlet channel should be used as the velocity and length scales. This is the usual hydraulic approach. The possible Reynolds number and non-dimensionalized position definitions are summarized as

$$Re^{\mathrm{A}} = 2u_{\mathrm{INLET}}h/v, \qquad X^{\mathrm{A}} = x/h,$$
$$Re^{\mathrm{B}} = 4u_{\mathrm{INLET}}h/vE, \qquad X^{\mathrm{B}} = x/h,$$
$$Re^{\mathrm{C}} = 2u_{\mathrm{INLET}}(H-h)/v, \qquad X^{\mathrm{C}} = x/(H-h),$$

where H is the height of the main channel and h is the height of the step as shown in Figure 1. When the expansion number is set to $E=2$, all the above definitions are equivalent. The various definitions were found to have little effect on the underlying behaviour when the expansion number was varied for a constant Reynolds number. For example, the percentage differences in the lengths x_1, x_2 and x_3 between the no-inlet-channel configuration and the flow configuration with a long inlet channel (fixed at $5H$) are summarized in Table III for $Re^{\mathrm{A,B,C}}=400$. The results indicate that the formation of the upper recirculation region is dependent on the expansion number. Below $E=2$ an upper recirculation region does not form for $Re=400$. Table III shows that the percentage difference in the reattachment and separation lengths decreases with higher expansion numbers no matter now the Reynolds number is defined. Smaller percentage differences for higher expansion numbers are a little surprising, because the dramatic change in geometry does not cause a dramatic change in fluid flow. In fact, depending on the Reynolds number definition, high-expansion-number results do have significant differences but they are strictly localized in the sudden expansion region, demonstrating that the viscous boundary condition from the walls quickly dissipates the entrance effect; this is discussed and quantified in the *AD* results below. (The Re^{C} results when the expansion number is $E=5$ are unsteady.)

The *AD* results in the inlet channel are dependent on the Reynolds number definition, although this is localized in the sudden expansion region. (The *AD* results are similar in behaviour to the maximum downward v-velocity results and the pressure coefficient results, so these last two terms are not discussed.) After a short distance upstream of the sudden expansion, $x \approx 0{\cdot}5H$, the underlying principle that the low expansion numbers are most affected by the sudden expansion is still valid. However, at the step the Re^{A} results predict that the deviation from the parabolic profile is greater for $E=5$ compared with $E=1{\cdot}25$. The Re^{B} results also predict that the deviation does not decrease with increasing expansion number at the sudden expansion. There is a greater deviation for the $E=5$ results compared with the $E=2{\cdot}5$ results, although the deviations in both cases are lower than the $E=1{\cdot}25$ results. The *AD* variations with entrance length are shown in Figure 6 for the Re^{B} results. For the higher expansion numbers the deviation from the parabolic profile does not occur significantly for entrance channel lengths of one or two step heights. The Re^{C} results, however,

Table III. Percentage difference between reattachment and separation lengths that use no inlet channel and a long inlet channel

E	X_1^{A}	X_2^{A}	X_3^{A}	X_1^{B}	X_2^{B}	X_3^{B}	X_1^{C}	X_2^{C}	X_3^{C}
1·25	12·34	—	—	14·28	—	—	17·80	—	—
1·667	6·45	—	—	6·98	—	—	7·57	—	—
2	4·61	5·95	2·23	4·61	5·95	2·23	4·61	5·95	2·23
2·5	2·94	3·30	2·22	2·64	3·15	1·95	2·52	3·06	1·68
5	0·95	0·75	1·67	0·86	1·04	0·53	*	*	*

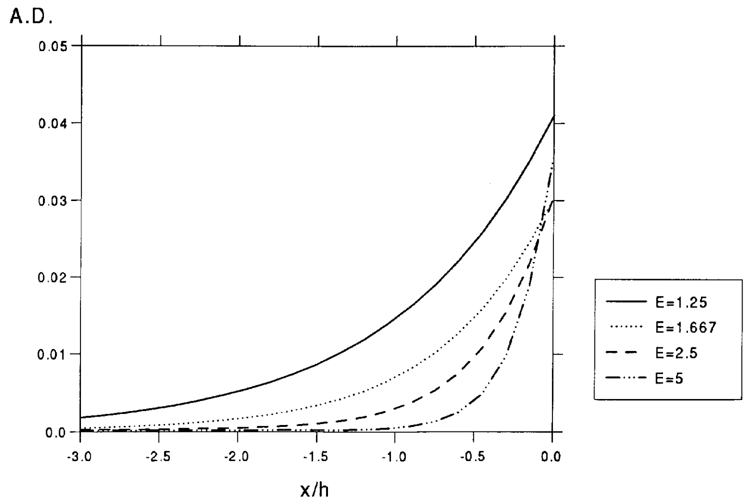

Figure 6. Variation in average deviation from parabolic profile with inlet channel entrance length

follow the simpler behaviour, i.e. the deviation from the parabola decreases with increasing expansion number in the whole domain.

The large *AD*-values for the Re^{A} and Re^{B} results are localized in the sudden expansion region. Upstream the viscous drag from the walls rapidly forces the *u*-velocity profile back into a parabolic profile. Downstream the deformed *u*-velocity profile either has a low inertia in the case of the Re^{A} results and the differences are readily dissipated or in the case of the Re^{B} results the *u*-velocity profile experiences a strong viscous action causing the differences again to become dissipated. The Re^{C} results have a large inlet velocity for high *E*-numbers and the flow is dominated by convection effects and therefore only low *E*-numbers are considerably affected by the sudden expansion.

CONCLUDING REMARKS

In conclusion, laminar flow over a backward-facing step has been numerically studied for a classical geometry. The study focused on the effect of using an inlet channel or specifying a commonly used inlet boundary condition at the sudden expansion.

It was found that when using an inlet channel, significant differences occur for low Reynolds numbers; however, they are localized in the sudden expansion region. In contrast, for high Reynolds numbers the main effect of the inlet channel was to shorten the main reattachment length and the upper separation length. The predictions that use an inlet channel appear to give better agreement with available experimental data where the data are most reliable.

Predictions with a uniform inlet velocity profile were used to investigate the boundary layer growth in the inlet channel. Differences in boundary layer thickness were only found near the sudden expansion region where the boundary layer was thicker along the upper boundary in comparison with the lower boundary.

Channel flows with low expansion numbers are most significantly affected by the entrance effect in the majority of the domain; the effect decreases exponentially with expansion number. However, depending on the velocity and length scales used, higher-expansion-number channels can also be considerably affected by the entrance effect, but in this case the differences are strongly localized in the sudden expansion region.

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